MATH 2263 SECTION 10 QUIZ 11

Name: ____________________________

Time limit: 15 minutes

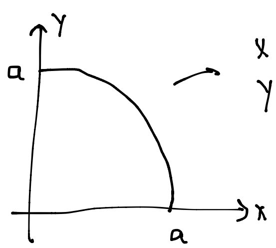
1. **(5 points)** Evaluate the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = \langle xy, 3y^2\rangle$ and C is given by the vector function $\mathbf{r}(t) = \langle t^4, t^3\rangle$, $0 \le t \le 1$.

$$\int_0^1 \langle t^7, 3t^6\rangle \cdot \langle 4t^3, 3t^2\rangle\, dt = \int_0^1 (4t^{10} + 9t^8)\, dt$$

$$= \left(4\cdot\frac{t^{11}}{11} + t^9\right)\Big|_0^1 = \frac{4}{11} + 1 = \frac{15}{11}$$

2. **(6 points)** A thin wire has the shape of the first-quadrant part of circle with center the origin and radius a. If the density function is $\rho(x,y) = kxy$, find the **center of mass** of the wire.

C:



$x = a\cos t$
$y = a\sin t$ $0 \le t \le \pi/2$.

So $m = \int_C \rho(x,y)\, ds = \int_0^{\pi/2} k\, a\cos t\cdot a\sin t\, \overbrace{\sqrt{(-a\sin t)^2 + (a\cos t)^2}}^{a}\, dt$

e: $\cos t\sin t = \frac{\sin 2t}{2}$

$= ka^3 \int_0^{\pi/2} \cos t\cdot\sin t\, dt \underset{\substack{u=\sin t\\ du=\cos t\, dt}}{=} ka^3\int_0^1 u\, du = \frac{ka^3}{2}$

ence $\bar{y} = \frac{1}{m}\int_C y\cdot\rho(x,y)\, ds = \frac{2}{\cancel{ka^3}}\cdot\cancel{ka^3}\int_0^{\pi/2} a\sin t\cdot\cos t\sin t\, dt = 2a\int_0^{\pi/2}\sin^2 t\cos t\, dt$

$u = \sin t$
$du = \cos t\, dt$

$= 2a\int_0^1 u^2\, du = \frac{2a}{3}$.

$\rho(x,y) = kxy$ and C are symmetric in x & y $\Longrightarrow$ so $\bar{x} = \frac{2a}{3}$ as well.

$\boxed{\text{Center of mass: } \left(\frac{2a}{3}, \frac{2a}{3}\right)}$

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3. **(4 points)** Match the vector fields $\mathbf{F}$ with the plots labeled I-IV. You **don't have to** justify your reasoning. (Note: The plots have been scaled for clarity.)

$\vec{F}(x,y) = \langle 2(x-y), -2(x-y) \rangle$

$\mathbf{F}(x, y) = \langle \sin(x + y), x \rangle$: III

$\mathbf{F} = \nabla f$ where $f(x, y) = (x - y)^2$: I

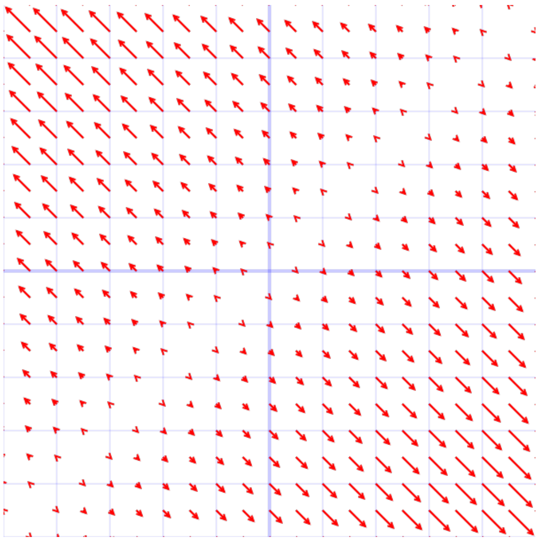
$\mathbf{F}(x, y) = \langle x + 2, x \rangle$: IV

$x^2 + xy$

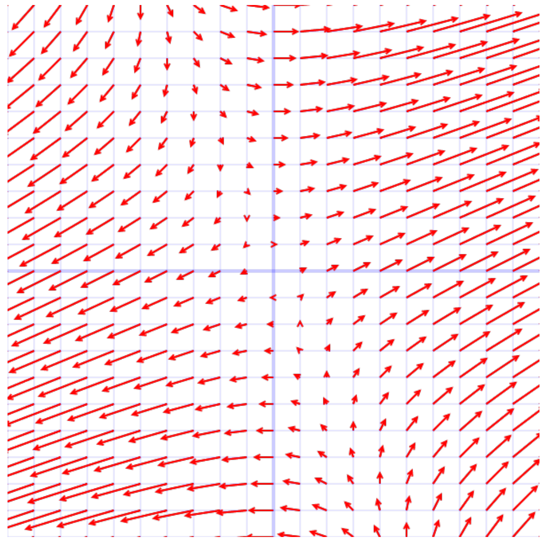
$\mathbf{F} = \nabla f$ where $f(x, y) = x(x + y)$: II

$\vec{F}(x,y) = \langle 2x+y, x \rangle$

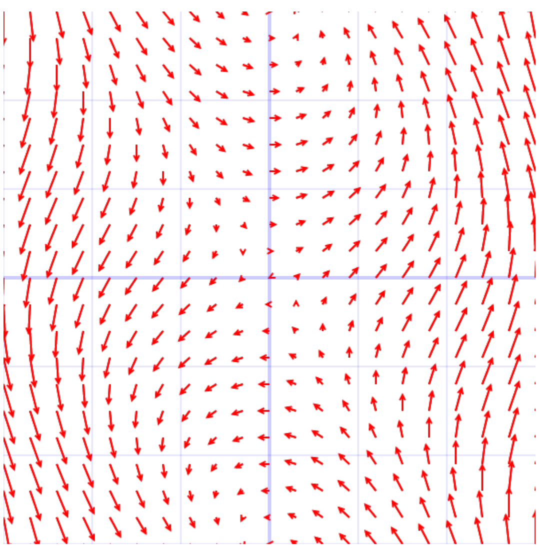
I



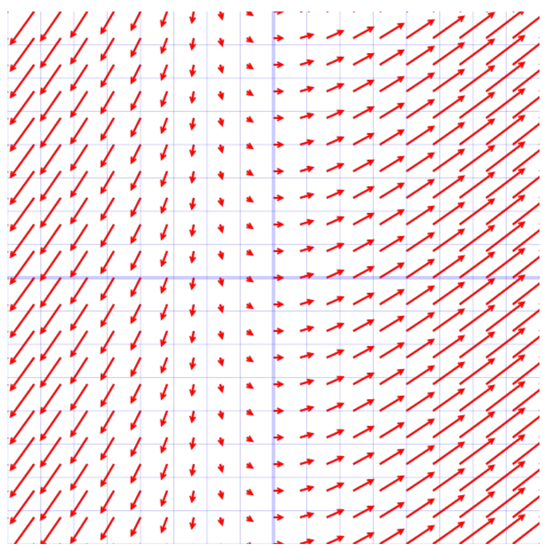
II



III



IV



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